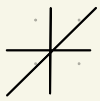
Absolute Values (Graphs & Sketching)

→ Denoted by $|\ |$, example → $|5|$, $|x+2|$, $y=|x|$

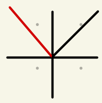
→ Easiest way to understand is that the graph below the x-axis gets reflected in the x-axis.

Examples

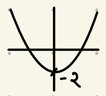
① $y=x$



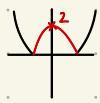
$y=|x|$



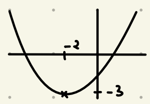
② $y=x^2-2$



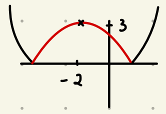
$y=|x^2-2|$



③ $y=(x+2)^2-3$



$y=|(x+2)^2-3|$



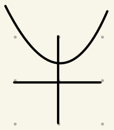
Easy way to draw reflected parabolas is by using the vertex. y-coordinate has to be positive

$(x,-y) \rightarrow (x,y)$

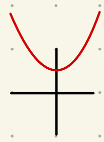
Note: If entire graph is above x-axis, then the absolute graph will have no change.

Example :

$y=x^2+2$



$y=|x^2+2|$



Reason: There are no negative y values therefore there is no change in the graph.

Piecewise Function

Definition → for $y=|x|$ graph

$$|x| \begin{cases} x\ ,\ x \geq 0 & ① \\ -x\ ,\ x < 0 & ② \end{cases}$$

In words: When x is equal or bigger than 0, y is equal to 0 ①

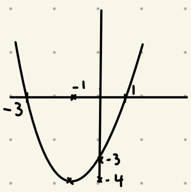
When x is less than 0, y is equal to negative x ②

Another way to think of it is having different boundaries for different parts of the graph.

Example : Sketch and find piecewise function for $y=|(x+1)^2-4|$

① Find vertex and intercepts for $y=(x+1)^2-4$. Vertex = $(-1,-4)$ Translated 4 units down and 1 unit left

② Sketch



x-intercept when $y=0 \rightarrow 0=(x+1)^2-4$

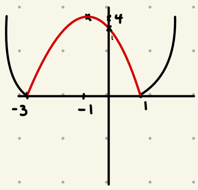
$4=(x+1)^2$

$\pm 2 = x+1$

$x=1\ ,\ x=-3$

y-intercept $= -3$

③ Sketch absolute



To create piecewise equation, check for what x-values the graph stays the same and for which it changes.

For $x \leq -3$ and $x \geq 1$, no change

For $-3 < x < 1$, flipped

$$\boxed{|(x+1)^2-4| \begin{cases} (x+1)^2-4, & x \geq 1\ \&\ x \leq -3 \\ -((x+1)^2-4), & -3 < x < 1 \end{cases}}$$