Differentiation

Examples

- Basic Rules

$\frac{dy}{dx} x^n = n \cdot x^{n-1} \rightarrow \frac{dy}{dx} x^3 = 3x^2$

$\frac{dy}{dx} e^{nx} = n \cdot e^{nx} \rightarrow \frac{dy}{dx} e^{3x} = 3e^{3x}$

$\frac{dy}{dx} \sin(n \cdot x) = n \cdot \cos(n \cdot x) \rightarrow \frac{dy}{dx} \sin 3x = 3 \cos 3x$

$\frac{dy}{dx} \cos(n \cdot x) = -n \cdot \sin(n \cdot x) \rightarrow \frac{dy}{dx} \cos 3x = -3 \cdot \sin 3x$

$\frac{dy}{dx} \ln(nx) = \frac{1}{x} \rightarrow \frac{dy}{dx} \ln(3x) = \frac{1}{x}$ Using chain rule

$\frac{dy}{dx} n = 0$ where n is a constant

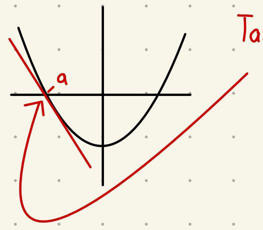
A common way to write derivatives is using an apostrophe instead of dy/dx.

↳ If $y \Leftrightarrow f(x)$ then $y' \Leftrightarrow f'(x)$

Slope

The slope of a tangent line at a point (a,b) on a curve can be found by deriving the curve's equation and subbing $x = a$.

↳ For curve $f(x)$, the slope of tangent at (a,b) is $m = f'(a)$.



Tangent to curve at $x = a$.

Rules

1. **Product Rule** → Used when 2 functions are multiplied

If $y = f(x) \cdot g(x)$, then $\frac{dy}{dx} = f'(x) \cdot g(x) + f(x) \cdot g'(x)$

Basically multiply each function with the derivative of the other.

$y = x^2 e^{3x}$

$f(x) = x^2$

$f'(x) = 2x$

$g(x) = e^{3x}$

$g'(x) = 3e^{3x}$

$\frac{dy}{dx} = 2x \cdot 3e^{3x} + x^2 \cdot 3e^{3x}$

2. **Quotient Rule** → Used when 2 functions are divided

If $y = \frac{f(x)}{g(x)}$, then $\frac{dy}{dx} = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{g^2}$, $g \neq 0$

Order matters.

$y = \frac{x^2+1}{x-1}$

$f(x) = x^2+1$

$f'(x) = 2x$

$g(x) = x-1$

$g'(x) = 1$

$\frac{dy}{dx} = \frac{(2x)(x-1) - (x^2+1)(1)}{(x-1)^2}$

3. **Chain Rule**

If $y = f(g(x))$, then $\frac{dy}{dx} = f'(g(x)) \cdot g'(x)$

Separate into inner and outer functions and differentiate separately. Use substitutions if necessary

$y = (2x^3 + 3x^2)^4$

Inner $= 2x^3 + 3x^2$

Outer $= (\quad)^4$

Ignore Inner Function

Inner$' = 3x^2 + 6x$

Outer$' = 4(\quad)^3$

$\frac{dy}{dx} = 4(2x^3+3x^2)^3 \cdot 3x^2 + 6x$