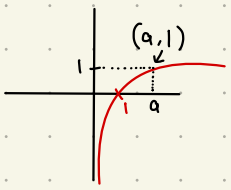
Logarithms & Graphs

Rules.

1. $a = \log_b c \rightarrow b^a = c$ [log and exponent form conversion]
2. For $\log_a b$, $a > 0$, $a \neq 1$
3. $\log_a(a^b) = b$
4. $a^{\log_a b} = b$, $b > 0$
5. $\log a = \log_{10} a$ [If no base specified, it is base 10]
6. $\log_a(bc) = \log_a b + \log_a c$ [Product Rule]
7. $\log_a\left(\frac{b}{c}\right) = \log_a b - \log_a c$ [Quotient Rule]
8. $\log_a(b^c) = c \cdot \log_a(b)$ [Power Rule]
9. $\log_a(b) = \frac{\log_c(b)}{\log_c(a)}$ [Change of Base]
10. $\log_a(1) = 0$
11. $\log_a(a) = 1$

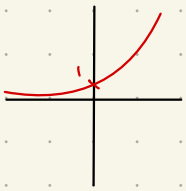
Log Graph



The normal $y = \log_a x$ graph has vertical asymptote $x = 0$ and x-intercept at $(1, 0)$.

All other graph transformations can be applied as normal.

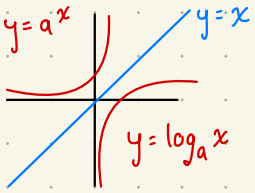
Exponent Graph



The normal $y = a^x$ graph has horizontal asymptote $y = 0$ and y-intercept at $(0, 1)$.

All other graph transformations can be applied as normal.

Relation between Log and Exponent Graph



The 2 graphs are inverses of each other.

Each graph is the other reflected in $y = x$.

Solving Common Log Problems [with laws used]

1. Single Log [Raise to log's power] (4)

$\log_3(2x-1) = 4 \rightarrow 3^{\log_3(2x-1)} = 3^4$

$\rightarrow 2x - 1 = 3^4$

2. Adding / Subtracting Logs (4) (6)

$\rightarrow \log_5(x+1) + \log_5(x) = 2$

$\rightarrow \log_5(x+1)(x) = 2$

$\rightarrow (x+1)(x) = 5^2$

3. Same Base

$\rightarrow \log_7(x+1) = \log_7(5)$

$x + 1 = 5$

4. Exponent Questions (8)

$\rightarrow 7^{2x-1} = 5$

$\log 7^{2x-1} = \log 5$ [Log both sides and then isolate x.]

$(2x-1)\log 7 = \log 5$

$2x - 1 = \frac{\log 5}{\log 7}$

$2^{x+1} = 4^{x+3}$ [Make the bases same]

$2^{x+1} = 2^{2(x+3)}$

$x + 1 = 2(x+3)$

5. Quadratic Logs

[Use a substition]

$(\log_2 x)^2 - 5\log_2 x + 6 = 0$

$u = \log_2 x$

$u^2 - 5u + 6 = 0 \rightarrow$ Find u then solve for x by solving $u = \log_2 x \rightarrow 2^u = x$