Math 10 Review

1. Exponent Laws

Factor Trees → Breaking numbers up into their prime factors.

Factors of 30 = 1, 2, 3, 5, 6, 10, 15, 30
Prime Factors of 30 = 2, 3, 5 (1 is not counted).

48 → 2 → 24 → 2 → 12 → 2 → 6 → 2 → 3 $\quad 48 = 2^4 \cdot 3^1$

60 → 2 → 30 → 2 → 15 → 3 → 5 $\quad 60 = 2^2 \cdot 3^1 \cdot 5^1$

Factor Trees

GCF = Greatest Common Factor or biggest factor that can divide the given numbers.

LCM = Lowest Common Multiple or smallest common multiple for given numbers.

For both GCF & LCM, you have to write numbers as a product of their prime factors.

$48 = 2^4 \cdot 3^1$
$60 = 2^2 \cdot 3^1 \cdot 5^1$

For GCF, only pick the common factors and the smallest power
$GCF = 2^2 \cdot 3^1 = 12$

$48 = 2^4 \cdot 3^1$
$60 = 2^2 \cdot 3^1 \cdot 5^1$

For LCM, pick all the factors and the largest power.
$LCM = 2^4 \cdot 3^1 \cdot 5 = 240$

Negative exponents → Turns a number into a reciprocal.

- $5^{-1} = \frac{1}{5}$
- $5^{-2} = \frac{1}{5^2}$
- $x^{-6} = \frac{1}{x^6}$
- $\frac{1}{x^{-7}} = x^7$ Double Reciprocal

Other Exponent Laws:

$x^2 \cdot x^3 = x^{2+3} = x^5$

$x^{10} \div x^8 = x^{10-8} = x^2 \quad // \quad \frac{x^{15}}{x^8} = x^{15-8} = x^7$

$(x^3)^7 = x^{3 \cdot 7} = x^{21}$

$25^0 = 1, \; 500^0 = 1, \; 7000^1 = 7000$

$\left(\frac{x^2}{y^2}\right)^4 = \frac{x^{2 \cdot 4}}{y^{2 \cdot 4}} = \frac{x^8}{y^8}$

2. Polynomials

Adding & Subtracting Variables
↳ You can add or subtract as long as same base and power. $x^2 + 3x^2 = 4x^2$
$2x^3 + 3x^4 = X$

Multiplying Variables → Any Variables can be multiplied
↳ $(x+3)(x+6) = x^2+6x+3x+18$ FOIL.

Factoring Trinomials where x^2 has no number with it.
↳ $x^2 + 11x + 30$ → Find 2 numbers that multiply to 30 and add to 11.

$5 \times 6 = 30$
$5 + 6 = 11$
$(x+5)(x+6)$

However when x^2 has a number, you need to check this way:

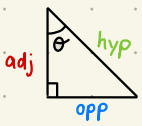
$2x^2 + 9x + 7$ → The 2 numbers must multiply to $2 \cdot 7 = 14$.

$2 \times 7 = 14$
$2 + 7 = 9$
$2x^2 + 7x + 2x + 7$
$x(2x+7) + 1(2x+7)$
$(x+1)(2x+7)$

Difference of Squares → $a^2 - b^2 = (a+b)(a-b)$
$y^2 - 25 = y^2 - 5^2 = (y-5)(y+5)$

$9x^2 - 100y^2 = 3^2x^2 - 10^2y^2 = (3x-10y)(3x+10y)$

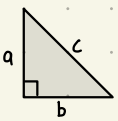
3. Trigonometry



$\sin\theta = \frac{opp}{hyp}$
$\cos\theta = \frac{adj}{hyp}$
$\tan\theta = \frac{opp}{adj}$

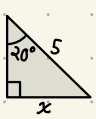
Only Works for right angled triangles

SOHCAHTOA



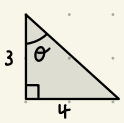
Pythagoras → $a^2 + b^2 = c^2$ OR think of it as the 2 shorter sides squared equalling to the longest side squared.

To find a side:



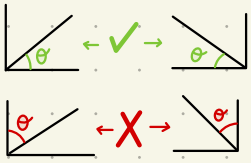
$\sin 20 = \frac{x}{5}$
$5 \cdot \sin 20 = x$

To find an angle:

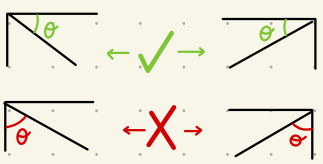


$\tan\theta = \frac{4}{3}$
$\theta = \tan^{-1}\left(\frac{4}{3}\right)$

Angle of Elevation:



Angle of Depression:



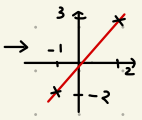
4. Functions & Relations

A graph is a function if it passes the **vertical line test**

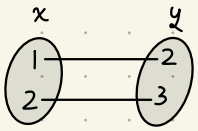
Must only cross once.

Equations can be graphed using **table of values** by **plotting corresponding** x & y value.

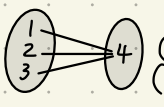
x	y
-1	-2
2	3



Function Notation is a way of **writing** x & y pairs.

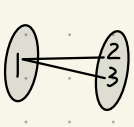


(1, 2)
(2, 3)

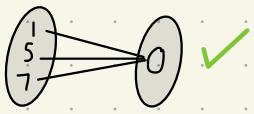


(1, 4)
(2, 4)
(3, 4)

Every x can **only have 1** y **value**, but y **values** can have **multiple** x**-values**.

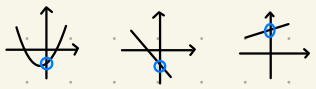


(1, 2)
(1, 3)
Not a Function

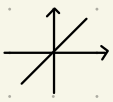


x-intercept is where function **cuts** x**-axis** or where $y = 0$

y-intercept is where function **cuts** y**-axis** or where $x = 0$

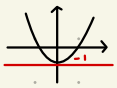


Domain → what x **values** can be. Usually $x \in \mathbb{R}$



No limits on x, x can be anything

Range → what y values can be. Usually an inequality.



$y \geq 1$

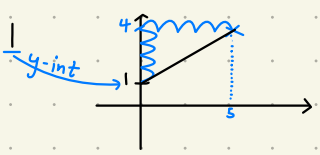
$y \in \mathbb{R}$

5. Linear Equations

General Form ⟶ $Ax + By = C$, ABC are integers
Slope Intercept Form ⟶ $y = mx + b$, m = slope, b = y-intercept
Point Slope Form ⟶ $y - y_1 = m(x - x_1)$, m = slope, (x_1, y_1) = point

Sketch using **Slope Intercept** since $m = \frac{rise}{run}$

$y = \frac{4}{5}x + 1$



$m = \frac{4}{5}$ → rise
→ run

Go 4 units up and 5 units right.

Slope

Positive Slope

Negative Slope

Parallel Lines have the **same slope**. $m_1 = \frac{2}{3}$ $m_2 = \frac{2}{3}$

Perpendicular lines have **negative reciprocal slope**.

$m_1 = 4$ $m_2 = -\frac{1}{4}$

6. Systems of Linear Equations

Solving linear means solving for x & y from given equations.

Substitution Method

$x = y + 1$ $\quad 2y = x + 1 \rightarrow 2y = (y+1) + 1$

$2y = y + 2$

$y = 2$

This method involves isolating 1 variable from an equation and substituting it into the other equation.

Addition / Elimination Method

$2x = 3y + 2$, $2x = 5 + 4y$

$$\begin{array}{r} 2x = 3y + 2 \\ -(2x = 5 + 4y) \\ \hline 0 = 7 + 7y \end{array}$$

$-7 = 7y$, $y = -1$

This involves either adding or subtracting equations to eliminate variables. You have to make sure one set of variables has the same coefficients so they cancel out.

If you get to a point where **all variables cancel** out and you are **only left with numbers**, you can have **2 scenarios**.

Infinite Solutions

$5 = 5$
$0 = 0$
→ Both Sides are equal

No Solutions

$1 = 2$
$2 = 3$
→ Both Sides are different

7. Arithmetic Sequences

3, 6, 9, 12, 15 ..... A sequence that is increasing by 3 and keeps going on forever.

+3 +3 +3 +3

n ⟶ For T_3 or 3rd term $n = 3$ and for 5th term $n = 5$
t_1 or a ⟶ First term
d ⟶ common difference, how much the terms increase or decrease by.

To find a term use $T_n = a + (n-1)d$ where n is the **number of term** you are trying to find.

The **sum** of **the first** n **terms** is S_n.

$S_n = \frac{n}{2} \cdot (2a + (n-1)d)$ or $S_n = \frac{n}{2}(a + t_n)$

t_n ↑ Last Term.