Reciprocal Graphs of Quadratic Functions

Definition → Reciprocal Graphs of quadratic functions can take 3 forms based on nature of roots [b^2-4ac → 2 roots, 1 root, no root]. General Form → $f(x) = \dfrac{1}{ax^2+bx+c}$

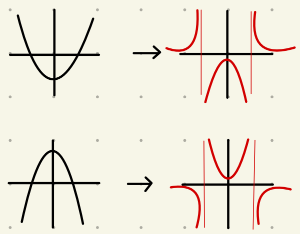
Important : Horizontal Asymptote for all forms is $y = 0$.

Case 1 : $b^2-4ac > 0$, 2 real roots (Normal Version)

Vertical Asymptotes (2) : Can be found by solving denominator set to 0. ($ax^2+bx+c = 0$)

Shape : 3 total curves, based on if original curve opens up (+) or down (-)

Vertex : For better sketches vertex can be found for the middle section by first finding $x = \dfrac{-b}{2a}$ then subbing that in the reciprocal equation to find y. (not really a vertex, more of a turning point)

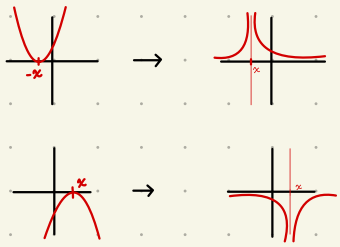


Case 2 : $b^2-4ac = 0$, 1 real root

Vertical Asymptote (1) : Once again can be found by solving denominator $= 0$.

Shape : 2 curves, based on if original is positive or negative

No vertex but y-intercept can be found with $x = 0$ substitution.

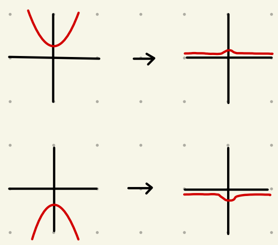


Case 3 : $b^2-4ac < 0$, No real roots

Vertical Asymptotes (0) : No vertical asymptotes, domain $x \in \mathbb{R}$

Shape : 1 curve, shape based on if original is positive or negative

Vertex : Turning point can be found same way using earlier method ($x = \frac{-b}{2a}$), and subbing into reciprocal equation.



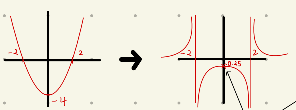
Examples :

Case 1 → $y = x^2 - 4$

Reciprocal → $y = \dfrac{1}{x^2-4}$

$x^2 - 4 = 0$

$x^2 = 4$, $x = \pm 2$ (Vertical asymptotes)



$x = 0 \rightarrow y = \dfrac{1}{0^2-4}$

$y = -0.25$

Case 2 → $y = (x-1)^2$

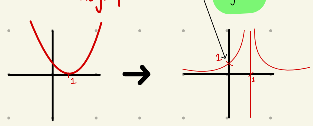
Reciprocal → $y = \dfrac{1}{(x-1)^2}$

$(x-1)^2 = 0$

$x = 1$ (vertical asymptote)

$x = 0 \rightarrow y = \dfrac{1}{(0-1)^2}$

$y = 1$



Case 3 → $y = x^2 + 3$

Reciprocal → $y = \dfrac{1}{x^2+3}$

$x^2 + 3 \neq 0$ (no vertical asymptote)

$x = 0$, $y = \dfrac{1}{0+3}$

$y = \dfrac{1}{3}$