Sequences & Series

Sequences and Series describe a bunch of terms in a sequence that are related to each other. There are 2 types.

- **Arithmetic Sequences**
  - ↳ Terms increase or decrease by a constant amount.
- **Geometric Sequences**
  - ↳ Terms increase or decrease by a common ratio.

Arithmetic

Example: $3+5+7\ldots\ldots 49+51\ldots$ (+2, +2, +2) → Terms differ by a **constant amount** (d). $d = a_n - a_{n-1}$

a = First Term
d = Constant Difference
n = number of terms

Term Formula

Used to find the n^{th} term (T_n).

$$T_n = a+(n-1)d$$

↳ To find 6th term use $n = 6$.

Sum Formula

Used to find the sum of the first n terms (S_n)

$$S_n = \frac{n}{2}(2a+(n-1)d)$$

– or –

$$S_n = \frac{n}{2}(a+T_n)$$

where T_n is the last term.

Sigma Notation

↳ Another way to represent the sum of terms.

$$\sum_{k=1}^{n} a+(k-1)d$$

(n: Final Term; $a+(k-1)d$: Formula; $k=1$: Starting Counter)

Example: $\sum_{k=1}^{3} 2k$

1st term = $2\cdot1 = 2$
2nd term = $2\cdot2 = 4$
3rd term = $2\cdot3 = 6$
Sum = $2+4+6 = 12$

Geometric

Example: $2, 4, 8, 16\ldots.\ 512, 1024\ldots.$ (×2, ×2, ×2, ×2)

$27, 9, 3, 1, 1/3, 1/9\ldots\ldots$ (×1/3, ×1/3, ×1/3, ×1/3, ×1/3)

→ Terms **increase** or **decrease** by a **common ratio** (r).
r is equal to a term **divided** by the **previous** term.

$$r = \frac{T_2}{T_1} = \frac{T_3}{T_2} = \frac{T_4}{T_3} = \frac{T_{n+1}}{T_n}$$ where $r \neq 1$.

Diverging Series: where $|r| \geq 1$, has **no finite sum**.

Converging Series: where $|r| < 1$, has **a finite sum** and a **formula** to **find it**.

Term Formula

Similar to Arithmetic, this is used to find the n^{th} term of a sequence.

$$T_n = a \cdot r^{n-1}$$

- a → First Term
- r → Common Ratio
- n → Term you're trying to find
  - ↳ For term 7 or T_7, use $n = 7$

Sum Formula

$$S_n = \frac{a(1-r^n)}{1-r}$$

– or –

$$S_n = \frac{a(r^n-1)}{r-1}$$

Sum to Infinity

For converging series, there is a finite or "max" sum since the terms get smaller and smaller.

$$S_\infty = \frac{a}{1-r}$$

Examples of Hard Questions

Arithmetic:

Given 2 terms, find a Sum.
↳ Find a and d from the 2 term equations then substitute into Sum equation.

If $a_4 = 17$ and $a_{12} = 49$, Find S_5

$a_4 \rightarrow 17 = a+(4-1)d$
$a_{12} \rightarrow 49 = a+(12-1)d$
↓
$a = 5, d = 4$
$S_5 = \frac{5}{2}[2\cdot5+(5-1)4]$

Geometric:

Given 2 terms, find r & a.
↳ Divide the 2 terms so that a cancels out and you can find r.

If $a_3 = 12$ and $a_8 = 384$,
$a_3 \rightarrow 12 = a\cdot r^{3-1}$
$a_{12} \rightarrow 384 = a\cdot r^{8-1}$

$\frac{a_{12}}{a_3} \rightarrow \frac{384}{12} = \frac{a\cdot r^7}{a\cdot r^2}$

$32 = r^5$, $r = 2$