Sketching Polynomials

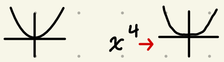
In order to sketch polynomials, you need →
1. Degree : Highest Power which controls shape
2. Leading Coefficent : Can flip graph with negative
3. Roots : Determines x-intercepts and y-intercepts
4. Multiplicity : Determines nature of roots

1. Degree

→ The overall shape of the graph depends on if the highest power is odd or even.

Even

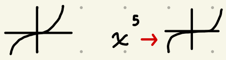
x^2 → x^4 →



General Shape will be a U.
The higher the power, the flatter it is.
Positive → QI & QII
Negative → QIII & QIV

Odd

x^3 → x^5 →



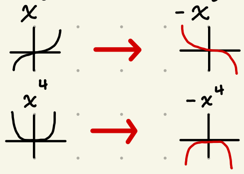
General Shape is a flowy curve
Has a tangent point, power ↑ flatter ↑
Positive → QIII & QI
Negative → QII & QIV

2. Leading Coefficent

If the leading coefficent has a negative sign, then the graph is flipped over the x-axis.

x^3 → $-x^3$

x^4 → $-x^4$



3 Roots

The roots or the x-intercepts are where the graph can touch, cut or tangent the x-axis. They can be found by putting the brackets equal to 0.

Example → $y=(x-3)^2(x+4)^2(x-7)$

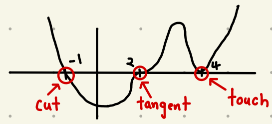
Roots → $x-3=0,\ x=3$
$x+4=0,\ x=-4$
$x-7=0,\ x=7$

→ (3,0), (-4,0), (7,0) are all points the graph touches/cuts the x-axis.

Full Examples

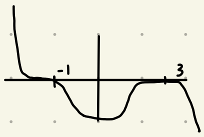
1. $y=(x-2)^3(x-4)^2(x+1)$

Highest degree $=x^6$ → even → QI & QII
Roots = 2 (tangent), 4 (touch), -1 (cut).



2. $y=-(x+1)^5(x-3)^4$

Highest degree $=-x^9$
Roots = -1, 3

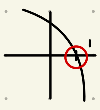


4. Multiplicity

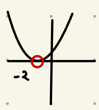
This determine's the nature of each root and how it interacts with the x-axis. There are 3 possible scenarios and it is based on the degree of the roots.

Example → $y=(x-1)(x+2)^2(x+3)^3$

1. Degree = 1.
The graph cuts the x-axis at the root only. (1,0)

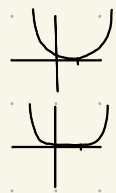


2. Degree = 2.
The graph touches the x-axis and rebounds without crossing



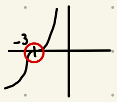
This is true for all even powers. The larger the power, the flatter the contact point.

$y=(x-1)^4$ →
$y=(x-1)^6$ →



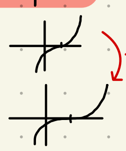
flatter

3. Degree = 3
The graph has a tangent point at the root and cuts the x-axis.



This is true for all odd powers. The higher the power, the flatter the tangent point.

$y=(x-1)^3$ →
$y=(x-1)^5$ →



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