Trig Basics

$\sin = \frac{opp.}{hyp.}$

$\cos = \frac{adj.}{hyp.}$

$\tan = \frac{opp.}{adj.}$

Degrees & Radians

1 revolution = 360° = 2π (180° = π)

Deg to rad → $\theta \times \frac{2\pi}{360}$

Rad to deg → $\theta \times \frac{360}{2\pi}$

Basic Angles

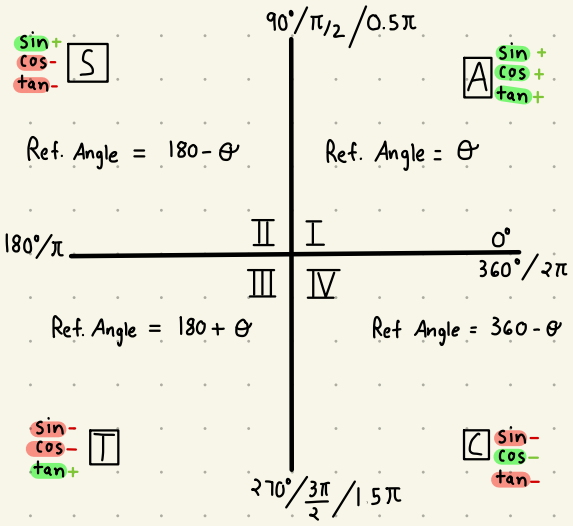
θ	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undef.

Tip: $\sin\theta = \frac{\sqrt{0,1,2,3,4}}{2}$

$\cos\theta = \frac{\sqrt{4,3,2,1,0}}{2}$

0, 30, 45, 60, 90

$0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$



Reciprocal Angles

$\text{cosec}\,\theta = \frac{1}{\sin\theta}$

$\sec\theta = \frac{1}{\cos\theta}$

$\tan\theta = \frac{1}{\tan\theta}$

eg: $\text{cosec}\,30 = \frac{1}{\sin 30} = 2$

Finding Reference Angles

→ Easiest way is to check how many revolutions by dividing 360 and then subtracting.

eg: 900°

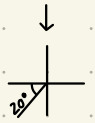
↳ $\frac{900}{360} = 2.5$ revolutions

900 - (2 × 360) = 180°

→ Use diagrams to find reference angle with x-axis for trig functions.

eg: sin 200 →

Therefore the reference angle for sin 200 would be sin 20 but since it is QIII, it would be negative



sin 200 = − (sin 20)

Infinite / General Solutions

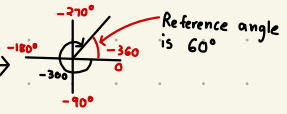
→ Trig Functions have infinitely many solutions as sin θ and cos θ repeat every 360° or 2π. tan θ repeats every 180° or π.

→ The number of answers for a trig function depends on the domain. If no domain given then there are infinite solutions.

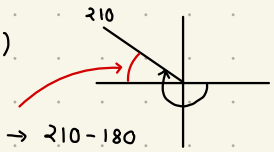
Negative Angles

→ Negative angles mean going in a clockwise direction instead of counterclockwise. The reference angles can then be found normally.

−300° →



eg: sin (−210)



Reference Angle → 210 − 180 = 30°

QII → sin positive so sin(−210) = sin(30) = ½

eg: Find all possible answers for $\cos\theta = 0.5$

cos θ = positive

- QI = θ
- QIV = 360 − θ

θ = cos⁻¹(0.5)

θ = 60°

Since no domain given, infinite solutions

Answer 1 → 60 + 360·n

Answer 2 → 300 + 360·n

Where n is a whole number positive or negative

General Form for infinite solutions or 2π·n for radians.

eg: Solve $\sin\theta = -\frac{\sqrt{3}}{2}$ for $-2\pi \le \theta \le 2\pi$

sin θ = negative

- QIII = π + θ
- QIV = 2π − θ

$\theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) \rightarrow \theta = \frac{\pi}{3}$

QIII answer → $\pi + \frac{\pi}{3} = \frac{4\pi}{3}$

QIV answer → $2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

Base Solutions.

Since domain is from −2π to 2π, we have to add and subtract 2π and check if it's in the domain

- $\frac{4}{3}\pi + 2\pi = \frac{10}{3}\pi \ge 2\pi$ ✗
- $\frac{4}{3}\pi - 2\pi = -\frac{2\pi}{3} \ge -2\pi$ ✓
- $\frac{5\pi}{3} + 2\pi = \frac{11\pi}{3} \ge 2\pi$ ✗
- $\frac{5\pi}{3} - 2\pi = -\frac{\pi}{3} \ge -2\pi$ ✓

Answers = $\frac{4\pi}{3}, \frac{5\pi}{3}, -\frac{2\pi}{3}, -\frac{\pi}{3}$